Name: KEY **Physics 233**
September 24, 2015 **Electron Spin**

1. Determine the vector: $|\Psi\rangle = |+z\rangle$. First, if the spin is in the +*z* direction, What are θ and ϕ?
   $\theta = 0, \phi = \text{undefined}$
   Then, determine a numerical value for the state vector.
   $$|\Psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos 0 \\ \sin 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2. Determine $|\Psi\rangle = |-z\rangle \quad \theta = \pi$
   $$|\Psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{2} \\ e^{i0}\sin\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

3. Determine $|\Psi\rangle = |+x\rangle \quad \theta = \pi/2, \phi = 0$
   $$|\Psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4} \\ e^{i0}\sin\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

4. Determine $|\Psi\rangle = |-y\rangle \quad \theta = \pi/2, \phi = -\pi/2$
   $$|\Psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4} \\ e^{-\pi i/2}\sin\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{bmatrix}$$

5. Determine the state vector if θ = 50.77° and ϕ = -90.
   $$|\Psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos 50.7 \\ e^{i\pi/2}\sin 50.7 \end{bmatrix} = \begin{bmatrix} 0.632 \\ -0.775i \end{bmatrix}$$

6. Determine the numerical value of $|\langle +z|+x\rangle|^2$.

$$\left| \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right|^2 = \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2}$$

7. Fill in the following table of probabilities.

$|\langle \text{out}|\text{in}\rangle|^2$	$|+z\rangle$	$|-z\rangle$	$|+x\rangle$	$|-x\rangle$	$|+y\rangle$	$|-y\rangle$
$\langle +z|$	1	0	½	½	½	½
$\langle -z|$	0	1	½	½	½	½
$\langle +x|$	½	½	1	0	½	½
$\langle -x|$	½	½	0	1	½	½
$\langle +y|$	½	½	½	½	1	0
$\langle -y|$	½	½	½	½	0	1

Run the SPINS simulation. The default experiment shows a quanton gun. You can add Carry out all of the experiments shown in the table above to determine if your calculations are correct. Show the instructor at least one of your runs, she will check off your worksheet.

Reset the Spins experiment (button on bottom, second from right). From the source menu, choose Unknown #1. Now, instead of a random collection of orientations, the Gun emits electrons in a definite state. You will determine this state.

8. Determine the probabilities of up and down in each of the three directions. Place your measurements in the following table.

Result	x-direction	y-direction	z-direction
Spin up ↑	½	0	½
Spin down ↓	½	1	½

9. Based on the above probabilities, what is state of the electrons released by the gun? Justify your answer.

   $|-y\rangle$

10. For unknown #2, we will begin with a prediction. This state is not along one of the three axes. Instead, the spin of these electrons makes and angle $\theta = 90$ with the z-axis and $\phi = 60$ with the x-axis, as in problem 5. Predict the probabilities in the table below.

$$|\langle\Psi|+y\rangle|^2 = \left| \begin{bmatrix} 0.632 & -0.775i \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ i\frac{1}{\sqrt{2}} \end{bmatrix} \right|^2 = |0.447+0.548|^2 = 0.99$$

$$|\langle\Psi|+z\rangle|^2 = \left| \begin{bmatrix} 0.632 & -0.775i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right|^2 = |0.632|^2 = 0.4$$

$$|\langle\Psi|+z\rangle|^2 = \left| \begin{bmatrix} 0.632 & -0.775i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right|^2 = |0.775|^2 = 0.6$$

	Prediction			Measurement		
Result	x-direction	y-direction	z-direction	x-direction	y-direction	z-direction
Spin up ↑	½	0.99	0.40	½	0.99	0.40
Spin down ↓	½	0.01	0.60	½	0.01	0.60

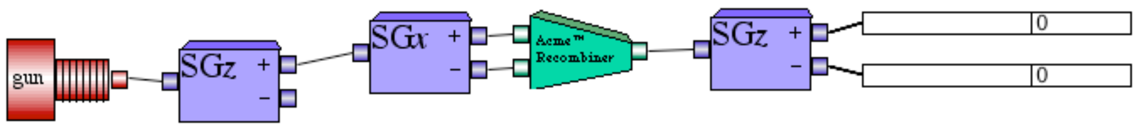
11. Make the measurements with the SPINS program and enter those probabilities above. Do they match what you expect? YES

12. Repeat steps 8 and 9 for unknown #4, which again is not aligned with the axes.

Result	x-direction	y-direction	z-direction
Spin up ↑	½	2/10	9/10
Spin down ↓	½	8/10	1/10

13. Assume that ϕ = -90. What is θ in this case? 18.4 degrees (based on asin(sqrt(0.1)))

Make an interferometer as shown:



14. Use `Unknown #1` as the initial state. Measure the relative probabilities of spin up and spin down after the final SG device. Do this for the three cases where (1) the spin up beam from the middle SG device is used, (2) the spin down beam from the middle SG device is used, and (3) both beams from the middle SG device are used (as the figure shows). Put your experimental results in the table on the worksheet. Repeat for the case where the spin down state is used from the first SG device.

Beams used		Experiment		Theory	
SG1	SG2	P ↑	P ↓	P ↑	P ↓
$\|+z\rangle$	$\|+x\rangle$	½	½		
	$\|-x\rangle$	½	½		
	$\|+x\rangle, \|-x\rangle$	1	0		
$\|-z\rangle$	$\|+x\rangle$	½	½		
	$\|-x\rangle$	½	½		
	$\|+x\rangle, \|-x\rangle$	0	1		

15. Now calculate the theoretically expected probabilities and fill in the theory part of the table. When both beams from the middle SG device are used, the input state to the last SG device (call it ψ) is a combination (superposition) of the spin up and spin down states with respect to the axis of the middle SG device. To calculate ψ properly, use the superposition rule (rule 5).